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Q1. What language does this grammar generate?

$$S \rightarrow aaS \mid A$$

$$A \rightarrow bbbA \mid \lambda$$

$L = \{a^n b^{3m} : n, m \geq 0\}$

$S \rightarrow aaS$
 $\rightarrow aaaS \rightarrow aaaaS \rightarrow \dots \rightarrow a^n A$
 $\rightarrow a^n bbbA \rightarrow a^n bbbbA \rightarrow \dots \rightarrow a^n b^{3m} A$
 $\rightarrow a^n b^{3m} \lambda = a^n b^{3m}$

which is formal lang L

Q2. Find a grammar for the following language.

$L = \{w \in \{0,1\}^* : w \text{ has an even number of 1's}\}$

$S \rightarrow 0, 1, 2, 3, 0, 11, 00, 1111, 0111, \dots$

$S \rightarrow w$
 \downarrow 0's between 1's
 Recursion

$S \rightarrow A11A \mid A \mid 0 \mid 0A \mid wA$

$A \rightarrow 11 \mid 1111 \mid \dots$

$S \rightarrow A1A1AS \mid \lambda$

$A \rightarrow 0A \mid \lambda$

University of Bahrain

Department of Computer Science

ITCS312 Automata and Formal Languages

ID:

Name:

KEY

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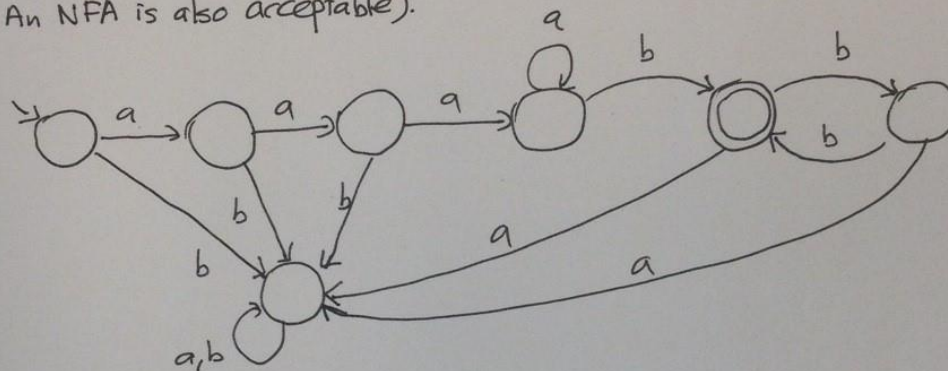
Quiz2

Problem 1.

Find a finite automaton for L given that

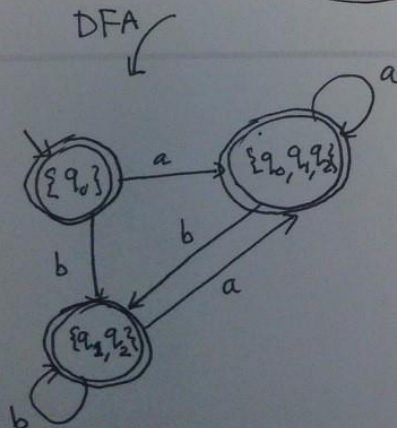
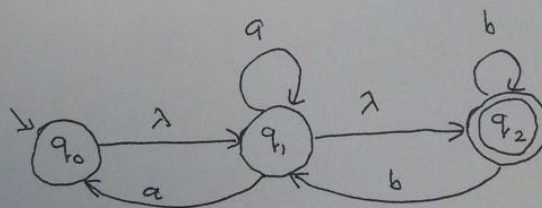
$$L = \{a^n b^m : n > 3 \text{ and } m \text{ is odd}\}.$$

DFA: (An NFA is also acceptable).



Problem 2.

Convert the following NFA to a DFA.



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Quiz 3 (Sections 3.1+3.2+3.3)

Q1. Find a regular expression for the following language. Assume that $\Sigma = \{a, b\}$.

$$L = \{ w \in \{a, b\}^* : n_a(w) \bmod 3 > 0 \}$$

Notice that the words in this language are not ordered in terms of a's and b's.

Answer: $r = (b^* a b^* a b^* a b^*)^* (a b^* + a b^* a b^*)$

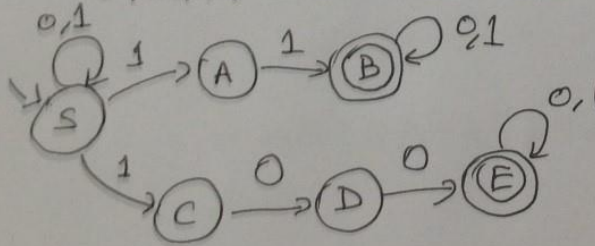
↑
this makes words with a's as a multiple of 3's

↑
followed by one or two a's to make it > 0

Q2. Find a regular grammar denoting the following language.

$$L = \{ w \in \{0, 1\}^* : w \text{ must contain the pattern } 11 \text{ or } 100 \}$$

First draw an NFA for the language



Now convert it to a right-linear grammar

$$\begin{aligned} S &\rightarrow 0S \mid 1S \mid 1A \\ A &\rightarrow 1B \\ B &\rightarrow 0B \mid 1B \mid \lambda \\ S &\rightarrow 1C \\ C &\rightarrow 0D \\ D &\rightarrow 0E \\ E &\rightarrow 0E \mid 1E \mid \lambda \end{aligned}$$

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Quiz4 (Take Home)

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Problem 1.

Show that given two regular languages L_1 and L_2 , then $L_1 - L_2$ is regular.

if L_1 is regular language & L_2 is regular language then \bar{L}_1 & \bar{L}_2 is also regular using the closure properties of regular languages.

also $L_1 \cap L_2 \rightarrow$ regular

\bar{L}_1 & $\bar{L}_2 \rightarrow$ regular

$\Rightarrow L_1 \cap \bar{L}_2 \Rightarrow$ is regular

Demorgan's Law :-

$$L_1 \cap L_2 = \overline{\bar{L}_1 \cup \bar{L}_2}$$

$$L_1 - L_2 = L_1 \cap \bar{L}_2$$

Problem 2.

Show that $L = \{a^n b^l c^k : k \neq n+1\}$ is not regular. $L_1 = \{a^n b^l c^k : k = n+1\}$

\Rightarrow

$$\text{Let } w = a^m b^m c^{2m}$$

$$|w| \geq m$$

$$w = xyz$$

$$\text{Since } |xy| \leq m \quad \& \quad |y| \geq 1$$

$$y = a^k \quad 1 \leq k \leq m$$

$$w_2 = xy^2z \Rightarrow a^{m+k} b^m c^{2m}$$

$$w_2 \notin L \Rightarrow 2m \neq (m+k) + m$$

$\therefore L_1$ is not regular using the pumping Lemma.

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Quiz # 5

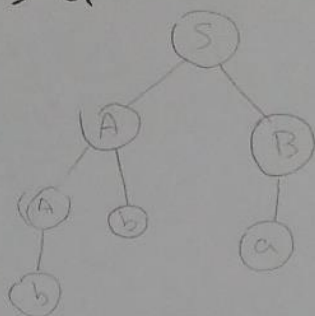
problem 1 :-

Show that the following grammar is ambiguous

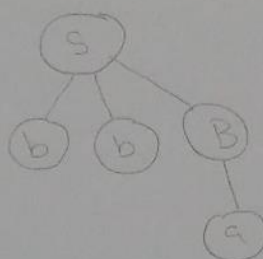
$$S \rightarrow AB \mid b b B$$

$$A \rightarrow b \mid A b$$

$$B \rightarrow a$$



$w = b b b a$



$w = b b a$

problem 2 :-

show that $L = \{a^n b^m c^k : 2n + m = k\}$ is context-free

$$S \rightarrow a a S c c \mid A$$

$$A \rightarrow \cancel{a} a b b \mid \cancel{b} B \mid \cancel{A} \mid b A c$$

$$B \rightarrow \cancel{c}$$

$$① S \rightarrow a a S c c \mid A$$

$$A \rightarrow b A c \mid \lambda$$

$$② S \rightarrow a a S c c \mid A$$

$$A \rightarrow b A c \mid \lambda$$

$$n=0$$

$$m=0$$

$$n=1 \quad m=1$$

$$2+1=3 \neq k$$

$$n=2 \quad m=2$$

$$4+2=6 \neq k$$

$$n=3 \quad m=1$$

$$6+1=7 \neq k$$

$$n=0 \quad m=1$$

$$0+1=1$$

$a a a a b b \lambda c c c$

$a a b c c c$

Q1. Remove all the unit, useless, and λ -productions from the following grammar.

$$S \rightarrow aSb \mid aA \mid B$$

$$A \rightarrow aC \mid \lambda$$

$$B \rightarrow aB \mid D$$

$$C \rightarrow a \mid aa \mid bC$$

$$D \rightarrow aBa$$

① Remove λ -prod.

$$V_n = \{A\}$$

$$S \rightarrow aSb \mid aA \mid a \mid B$$

$$A \rightarrow aC \mid a$$

$$B \rightarrow aB \mid D$$

$$C \rightarrow a \mid aa \mid bC$$

$$D \rightarrow aBa$$

② Remove unit-prod.

$$S \rightarrow B \rightarrow D$$

$$S \rightarrow aSb \mid aA \mid a \mid aB \mid aBa$$

$$A \rightarrow aC \mid a$$

$$B \rightarrow aB \mid aBa$$

$$C \rightarrow a \mid aa \mid bC$$

$$D \rightarrow aBa$$

③ Remove useless prod containing

D - unreachable

B - endless

$$S \rightarrow aSb \mid aA \mid a$$

$$A \rightarrow aC \mid a$$

$$C \rightarrow a \mid aa \mid bC$$

Q2. Convert the following context-free grammar to Chomsky Normal Form.

$$S \rightarrow \cancel{X}a\cancel{S}b \mid \cancel{X}a\cancel{X}Sb \mid \cancel{X}a\cancel{X}a\cancel{S}b \mid \cancel{X}A\cancel{X}b$$

$$A \rightarrow \cancel{X}b$$

step 1. $X_a \rightarrow a$

$$X_b \rightarrow b$$

$$S \rightarrow X_aB \mid CD \mid CE \mid b$$

step 2.

$$A \rightarrow b$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

CNF

$$B \rightarrow SX_b$$

$$C \rightarrow X_aX_a$$

$$D \rightarrow SX_b$$

$$E \rightarrow X_aD$$

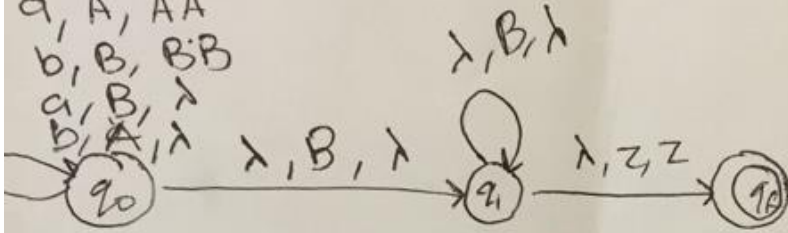
Quiz 7

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Problem 1.

Build an NPDA for the language $L = \{ w \in \{a,b\}^* : n_a(w) < n_b(w) \}$.

q, z, AZ
b, z, BZ
a, A, AA
b, B, BB
a, B, A
b, A, B



baabbb

abbb

abbb

abbb

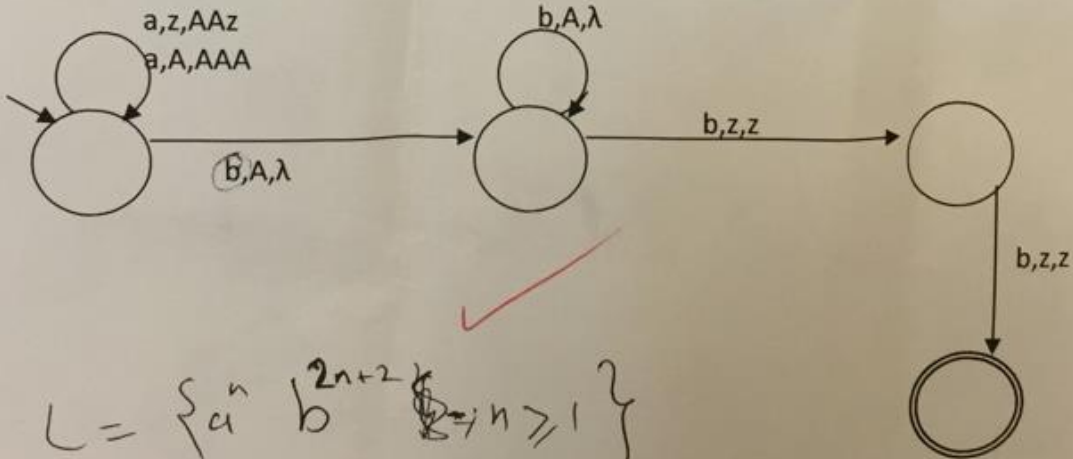
abb

ab

ab

Problem 2.

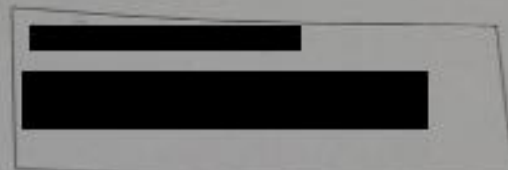
What is the language accepted by the following NPDA?



$$L = \{ a^n b^{2n+2} \mid n \geq 1 \}$$

aaabbbbbb
a

3 6+
a

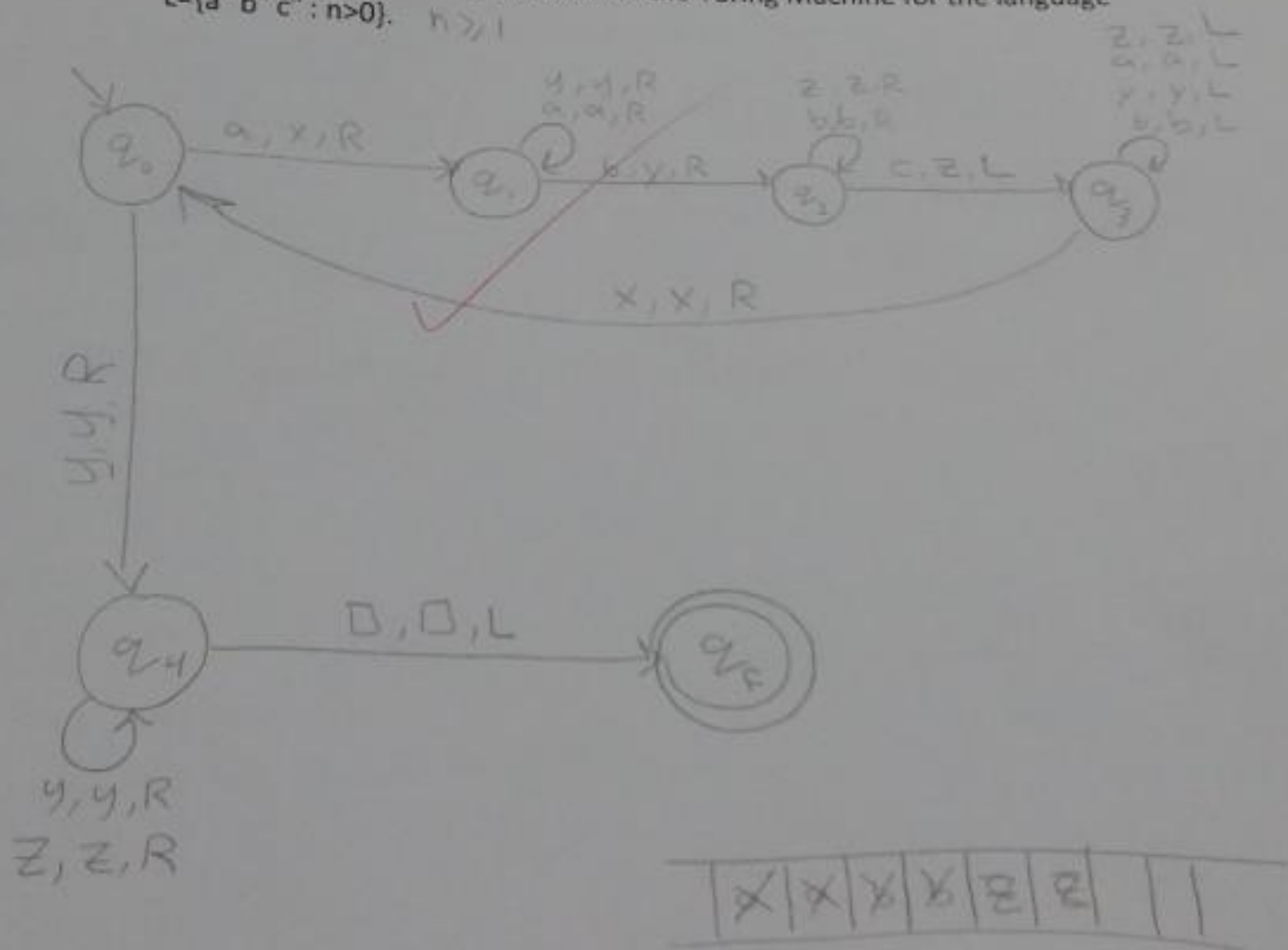


Quiz 9

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Problem 1.

Construct the transition function for the Turing Machine for the language
 $L = \{a^n b^n c^n : n > 0\}$. $n \geq 1$



Show that the function $f(x) = 3x$ is Turing-computable over the domain of the positive integers.

